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Mathematical modeling of optimal tilt angles of solar collector and sunray reflector

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In this paper, we present the mathematical modeling of the optimal angles of a solar collector and a solar collector with a sunray concentrator. The quantity of solar energy depends on geographical position, the trajectory of the sun, on the intensity of solar radiation energy, of sunlight duration per day and per year, the reflection coefficient of SRR, etc. Therefore, in different seasons, we will have different optimal angles of a solar collector. In Lithuania, the optimal angle of tilt of a solar collector is determined experimentally is 15° to 60° for the whole year: 15°...45° in winter and 30°... 60° in summer.

The mathematical model presented in this paper gives the possibility to calculate the optimal angles of tilt of a solar collector and a solar collector with a sunray reflector in a selected geographical position, the collector exploitation period, the geometrical parameters of a solar collector and a sunray reflector, etc. A comparison of the mathematical modeling and experimental data are presented and conclusions are made.
Key words: solar collector, reflector, solar ray reflector, optimal angles, solar radiant energy, mathematical modeling

1. INTRODUCTION

The use of renewable energy sources has a positive influence on the environmental pollution and global warming processes. The dependence on expensive traditional energy sources and the necessity to economize them promoted the creation and development of new technologies of ecological energy systems. One of them is solar energy.

At present, solar energy takes a small part of the energy market. However, the implementation of modern technologies can considerably change the situation, and possibilities of solar energy application will be more extensive.

Orientation of solar collector (SC) in space is the main factor influencing the quantity of absorbed solar radiation energy [1–3]. In the case with optimal angles of a solar collector, we will have the maximum of solar radiant energy.

The optimal angles depend on the geographical position and on the investigation period (day, week, month, etc.) when the position of a solar collector will be stationary. It is very important, because the trajectory of the sun changes. The intensity of solar radiation energy, sunlight duration per day and per year change as well.

Another way to increase the quantity of solar radiation energy is the use of a sunray concentrator-reflector (SRR). The main purpose of all sunray concentrators is to increase the quantity of solar radiation energy by using cheap materials and equipments of energy concentration. There are a lot of scientific researches on different shapes (parabolic, luminescent, etc.) of sunray concentrators-reflectors [4, 5]. In each case, selection of the type of a solar collector and concentrator system depends on the purpose. A flat reflector is most efficient for electricity and hot water production. The use of a plane reflector can increase the quantity of solar radiation energy by about 30% to 40% and sometimes even 50% to 60% [1, 6–8]. The material of a sunray concentrator is also very important [8].

A revue of literature sources shows that the modeling of solar position, the duration of sunlight, the intensity of solar radiation energy, the cross-section of solar rays falling into a solar collector (without reflector) depend on time [9, 10].

In this paper, we present a mathematical model designed to find the optimal angles of a solar collector, a solar collector with a sunray reflector and the results of an experimental investigation. We have estimated the sun position, sunlight duration per day and per year, the geographical position and geometrical parameters. In the case of a solar collector with a sunray reflector, both parameters – the cross-section of solar rays falling into a solar collector and the cross-section of solar rays falling into a sunray reflector – have been estimated.

2. MATHEMATICAL MODELING OF OPTIMAL ANGLES OF SC AND SC WITH SRR

There are many factors on which the quantity of solar radiation energy W (kWh / m^2) per time, falling into a SC depends.

All these factors can be distributed into two groups: ordinary factors and selected factors. The group of selected factors consists of the geometrical parameters of SC and of SC with SRR (length N (m), width B (m), height L (m)), angles β and γ which describe the orientation of SC and SC with SRR, latitude

Ψ, height above sea level, the reflection coefficient of SRR *H* and the investigation period *D*.

The group of ordinary factors consists of angles α and φ which describe the position of the sun in space, the intensity of solar radiation energy *Q* (*W*), the duration of sunlight. All these ordinary factors depend on time. They are described by known mathematical functions or are obtained from experimental data for a long period.

In this mathematical model, only selected factors β and γ are variable. All other ordinary factors depend on the geographical position, investigation period, etc. They are constant in a certain situation.

In a usual case of SC orientation, we will search for the maximum W_{max} of the function $W = f(\gamma)$. In case of SC with SRR, we will search for the maximum W_{max} of the function $W = f(\beta, \gamma)$. Thereby, we will get the optimal orientation angles β and γ as a result of calculations.

The numerical values of selected factors are presented in Table 1.

Table 1. **Selected factors and their numerical values**

Selected factors	Notation	Numerical value
Height of SC	*L*	1 m
Length of SRR	*N*	2 m
Width of SC and SRR	*B*	3 m
Latitude	ψ	55°44' N
Height above sea level	*A*	50 m
Reflection coefficient of SRR	*H*	0.95
Investigation period	*D*	60th day of the year

Note. The normal of the surface plane of SC and SC with SRR is oriented towards the south.

Calculations of the absorbed quantity of solar radiation energy *W* are made in the following conditions: β = 0°, γ = 90°.

The function $Q = f(t)$ multiplied by the function $A = f(t)$ gives the quantity of solar radiation energy *W* falling into SC per time *t*.

The total cross-section area *A* of sunrays falling into SC is calculated:

$$A = A_{SC} + A_{SRR} = A_{SK} + A_{SKK}, \quad (1)$$

where A_{SC} is the cross-section area of sunrays falling directly into the SC surface, m², and A_{SRR} is the cross-section area of solar rays reflected from the SRR surface and falling on the SC surface area, m².

There are five equations for calculating the total cross-section area A_{SRR} of solar rays falling into the SRR (Table 2).

The functions $A_{SC} = f(t)$ and $A_{SRR} = f(t)$ are calculated by using equation (2) and equations presented in Table 2.

$$A_{SC} = BL \sin(180 - \alpha - \gamma) \cos \phi. \quad (2)$$

The function $A = f(t)$ is the sum of the functions $A_{SC} = f(t)$ and $A_{SRR} = f(t)$, where *B* is the width of the solar ray concentrator (SRR) and the solar collector (SC), *m*,

L is the length of the solar collector (SC), m,

α is the angle between solar rays and the horizon, degrees,

γ is the angle between the solar collector and the horizon, degrees,

φ is the angle between solar rays and the south, degrees.

The coefficients α', φ', *E* and *P* are calculated:

Table 2. **Equations for calculating cross-section area A_{SRR} of solar rays falling into SRR**

No.	Condition	Equation
1.	α – β < α', φ < φ'	$A_{SRR} = BN \sin(\alpha - \beta)(1 - \frac{N}{2B} tg\varphi)$ (2)
2.	α – β < α', φ > φ'	$A_{SRR} = \frac{B^2}{2} \sin(\alpha - \beta) tg(90 - \varphi)$ (3)
3.	α – β > α', φ < φ'	$A_{SRR} = BL \sin(2\beta + \gamma - \alpha)(1 - \frac{P}{2B} tg\varphi)$ (4)
4.	α – β > α', φ > φ', *E* > *P*	$A_{SRR} = \frac{B^2 L}{2P} \sin(2\beta + \gamma - \alpha) tg(90 - \varphi)(1 - \frac{(E-P)^2}{E^2})$ (5)
5.	α – β > α', φ > φ', *E* < *P*	$A_{SRR} = \frac{B^2 L}{2P} \sin(2\beta + \gamma - \alpha) tg(90 - \varphi)$ (6)

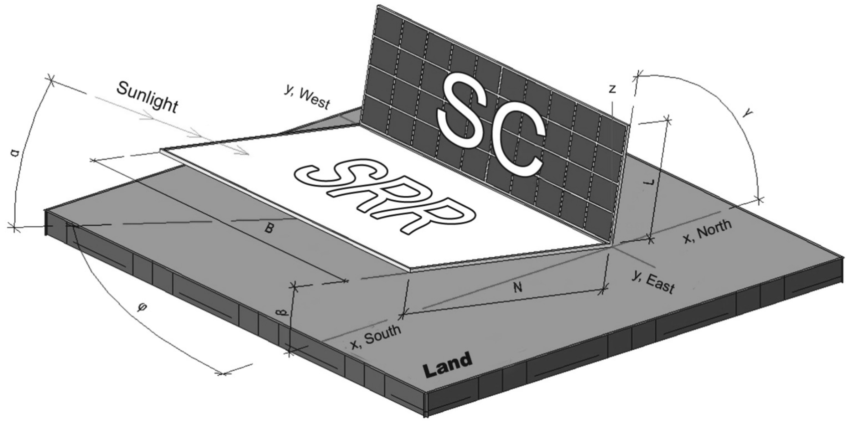

Fig. 1. SC and SRR geometrical parameters *N*, *B*, *L* and angles β and γ which describe SC and SRR orientation. Angles α and φ describe the position of the sun

$$\alpha' = \arccos\left(\frac{N - L\cos(180-\beta-\gamma)}{\sqrt{N^2 + L^2 - 2NL\cos(180-\beta-\gamma)}}\right), \quad (3)$$

$$P = \frac{L\sin(2\beta-\alpha+\gamma)}{\sin(\alpha-\beta)}, \quad (4)$$

$$E = \mathrm{Btg}\,(90-\varphi)\cos\beta, \quad (5)$$

$$A_{\mathrm{SRR}} = BL\sin(2\beta+\gamma-\alpha)(1-\frac{P}{2B}tg\varphi). \quad (6)$$

The dependence between angles α and φ characterizing solar position and the intensity of solar radiation energy Q vs. time is calculated by the method presented in [9].

By multiplying the functions $Q = f(t)$ and $A = f(t)$, we obtain the quantity of absorbed solar radiation energy W per time t, i. e. the function $W = f(t)$. The functions $W_{SC} = f(t)$ and $W_{SRR} = f(t)$ are components of W (Fig. 2).

The quantity of solar radiation energy falling into a SC with a SRR per day is calculated by integrating the equation $W = -9.874t^2 + 236.9t - 900.4$, i. e. the quantity of solar energy is equal to the area of a figure within the limits of the segment $[a\ b]$ and the function $W = f(t)$. The segment $[a\ b]$ describes the duration of solar radiation energy falling into a SC; here, a is the start point and b is the end point. The quantity of solar radiation energy W_{SC} falling into a SC is calculated in the same way.

First, we have to find the integrated limits $[a, b]$, i. e. the time interval t (hour) when solar radiation energy falling into the SC (a – time (hours), when the solar rays start to fall into the collector, b – time (hours) when solar rays stop falling into the collector).

The SC under consideration is stationary (one time orientation) in the general case, and solar radiation energy falls into the solar collector when:

- angle $\varphi < 90$ – a period when the angle between the south and the sun is less than 90°;
- angle $\alpha < 90$ – a period when the sun is above the horizon.

The function $\alpha = f(t)$ is calculated by the method presented in [9].

The integral is calculated within the limits $t = [6.863; 17.137]$:

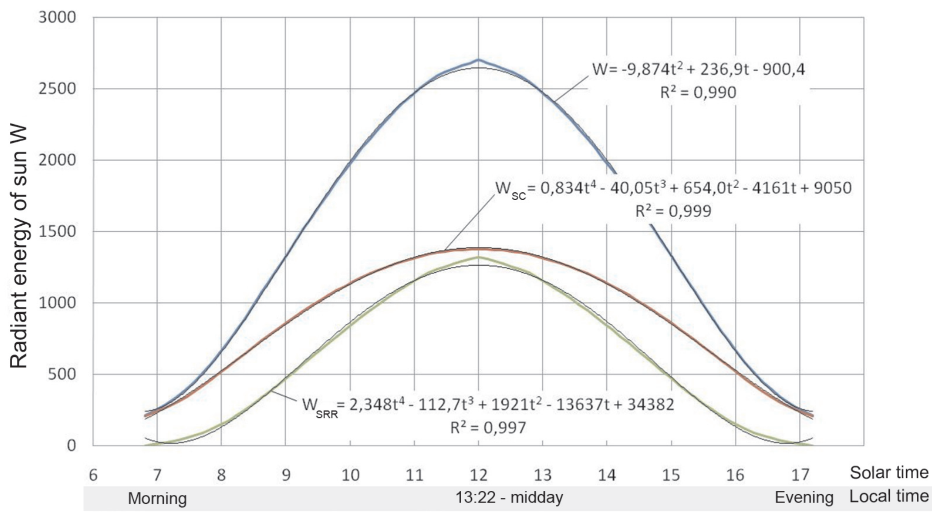

Fig. 2. Functions of the quantity of solar radiation energy W falling into SC, SRR and SC with SRR vs. time in selected conditions (Table 1)

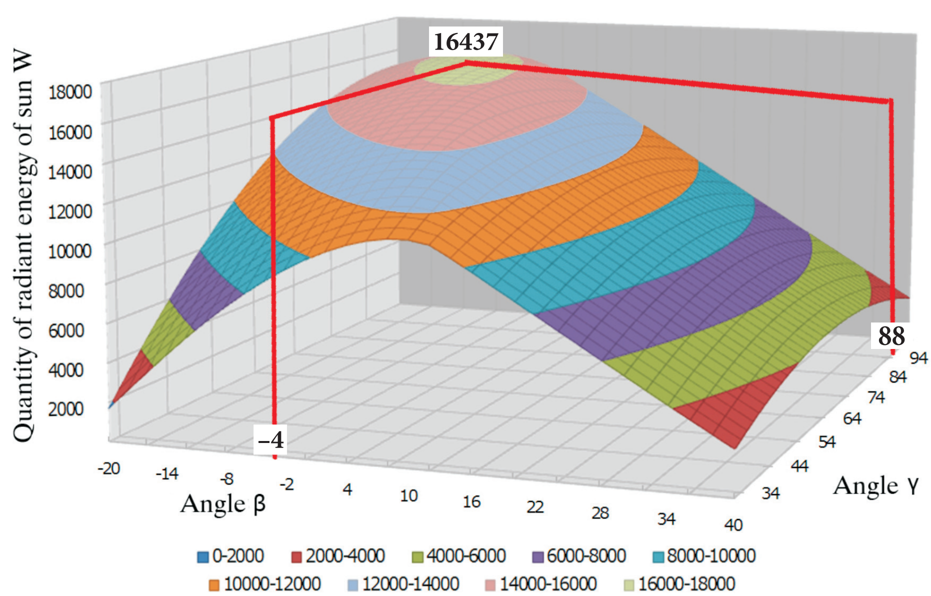

Fig. 3. Diagram of the function $W = f(\beta, \gamma)$

$$W = \int_{6.863}^{17.137} (-9.874t^2 + 236t - 900.4)dt = \\ = 15.99 \text{ kWh/m}^2 \text{ day}, \quad (7)$$

$$W_{SC} = \int_{6.863}^{17.137} (0.834t^4 + 40.05t^3 + 654t^2 - 4161t + 9050)dt = \\ = 9.426 \text{ kWh/m}^2 \text{ day}. \quad (8)$$

Therefore, during the 60th day of the year in the selected conditions, the quantity of solar radiation energy falling into a SC with a SRR will be W = 15.99 kWh/m^2 day when $\beta = 0$ and $\gamma = 90$. When $\gamma = 90$, W_{SC} = 9.426 kWh/m^2 day.

In order to find out the optimal angles β and γ in the case of a SC with a SRR, we have to calculate the available quantity of solar radiation energy with different values of angles β and γ. As a result, we will fill up the matrix of the function $W = f(\beta, \gamma)$.

In the case when we used a solar collector without an SRR, the quantity of solar radiation energy was calculated with different values of the angle γ, i. e. the function $W_{SC} = f(\gamma)$ was calculated.

It is accepted that the permissible range of the arguments $\beta = [-20; 50], \gamma = [-30; 110]$.

As a result of data matrix approximation, we identified that the function $W = f(\beta, \gamma)$ had the maximum value W_{max} = 16437 when $\beta = -4$, $\gamma = 88$, and the maximum of the function $W_{SC} = f(\gamma)$, W_{SCmax} = 10107, was when $\gamma = 68$.

The diagrams of the functions $W = f(\beta, \gamma)$ and $W_{SC} = f(\gamma)$ were compiled using the results of the data matrix (Figs. 3, 4).

According to the results of theoretical calculations with optimal angles β and γ, in the case of a SC with a SRR, the quantity of absorbed solar radiation energy W is by 62.6% higher than the quantity of absorbed solar radiation energy W_{SC} in a SC without a SRR.

In order to calculate the optimal angles β and γ for a selected period (week, month or year), we have to find the values of the functions $W = f(\beta, \gamma)$ and $W_{SC} = f(\gamma)$ for each day of the period.

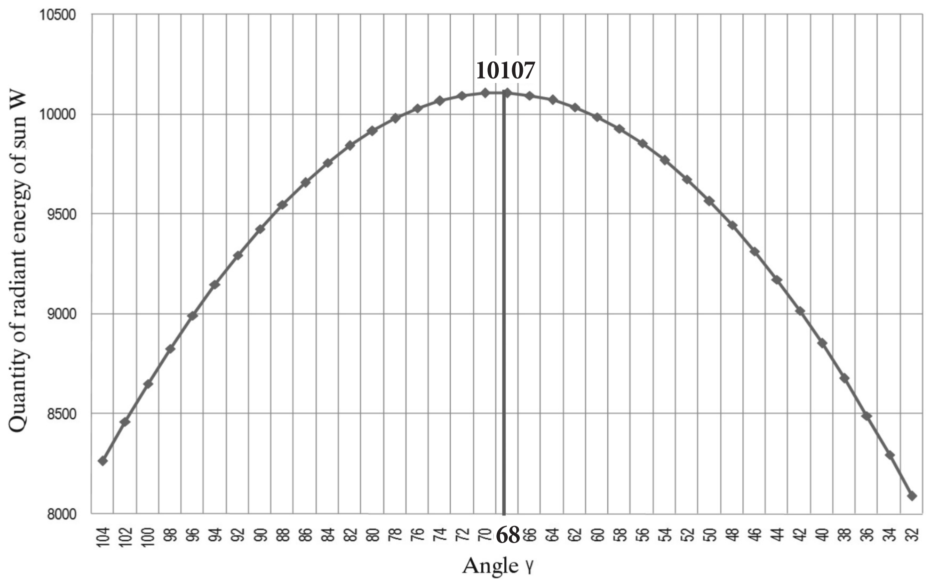

Fig. 4. Diagram of the function $W_{SC} = f(\gamma)$

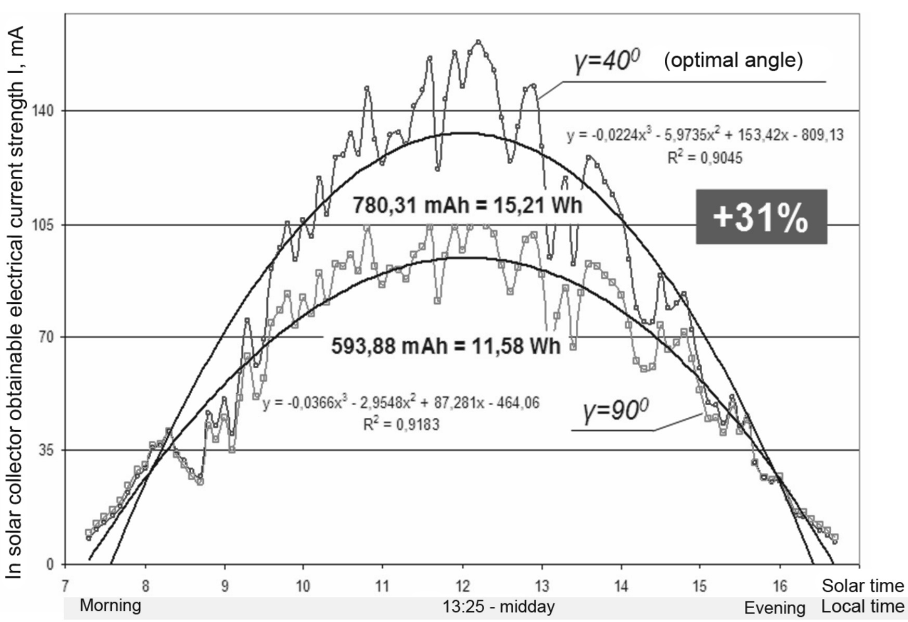

Fig. 5. Electric current generated by solar collectors (orientation angles $\gamma = 40°$ and $\gamma = 90°$) vs. time (experimental data)

After summing up the functions we obtained the data matrix and, as a result of its approximation, the maximum values W_{max} and W_{SCmax} were identified for a selected period.

3. EXPERIMENTAL

The application of the proposed mathematical model gives us a possibility to make theoretical calculations in cases of a SC and a SC with a SRR using the optimal angles γ, β and any freely selected angles. Then we will compare theoretical results with each other and with experimental data.

We selected the 14th July of for our experiment (the 143th day of the year). The nebulosity on that day was about 20%. A solar collector type T-SM2 (135 × 318 × 2 mm, I = 230 mA) (JSC "Saulės energija") a UT33 digital tester and a compass were used.

The results of applying the mathematical mode presented in the previous chapter have shown that the optimal angle of solar collector orientation is γ = 40° for the 143rd day of the year. The quantity of absorbed solar radiation energy is $W = 113.49$ (Wh). Using any freely selected angle, for example γ = 90°, the quantity of absorbed solar radiation energy is $W = 89.59$ (Wh).

A comparison of mathematical modeling results shows that SC orientation at the optimal angle γ = 40° gives a 26% better result than a SC with the angle γ = 90°.

The experiment was performed in the same conditions as the mathematical modeling, i. e. γ = 40° and γ = 90°. The collectors were oriented to the south. The electric current (*mA*) was registered in solar collectors every six minutes with a tester. Naturally the electric current was generated in a solar collector in proportion to the solar radiation energy falling on the SC surface [11]. For that reason, the electric current generated in a solar collector can be accepted as an equivalent of solar radiation energy. The results of the investigation are presented in Fig. 5.

As a result of the third level polynomial approximation of the experimental data, the function equations of the study results were defined:

for γ = 40°: $y = -0.0224x^3 - 5.9735x^2 + 153.42x - 809.1$,

for γ = 90°: $y = -0.0366x^3 - 2.9548x^2 + 87.281x - 464.0$,

where y is the generated electrical current, mA;

x is time, hours.

The results of integration show that the generated electric current is $I_{40} = 780.31$ mA when the SC orientation angle is optimal (γ = 40°) and gives a 31% better result than a SC with the angle γ = 90° ($I_{90} = 593.88$ mA).

A comparison of the experimental and mathematical modeling results for the 143th day of the year shows a 16% difference between the experimental and the mathematical data. The mathematical modeling results quite well correspond to experimental data and could be applied as a tool for finding the optimal angles.

For a more exact comparison of experimental and theoretical data, the experiment should be performed with more than two solar collectors with different orientation angles at the same moment of time.

4. CONCLUSIONS

1. Mathematical modeling gives us a possibility to calculate the optimal orientation angles β and γ of a solar collector and a solar collector with a solar ray concentrator under different conditions such as geographical position, geometric parameters of a SC and a SC with a SRR, reflection coefficient, selected period of time, etc.

2. To calculate the optimal orientation angles of a SC for a period longer than one day, it is necessary to estimate sunshine duration per year according to perennial meteorological data.

3. According to the results of mathematical modeling of the 60th day of the year with the optimal angles β and γ, in the case of a SC with a SRR, the quantity of absorbed solar radiation energy W was by 62.6% higher than the quantity of solar radiation energy W_{SC} absorbed in a solar collector with out a SRR. In another day of the year or in another selected period, the result of mathematical modeling can be different.

4. A comparison of the results shows a 16% difference between experimental and mathematical data. The results of mathematical modeling quite well correspond to experimental data and could be applied as a tool for finding the optimal angles.

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SAULĖS KOLEKTORIAUS IR SAULĖS SPINDULIŲ REFLEKTORIAUS OPTIMALIŲ POSVYRIO KAMPŲ MATEMATINIS MODELIAVIMAS

Santrauka

Nagrinėjamas saulės kolektorių ir saulės kolektorių su saulės spindulių koncentratoriumi optimalių orientavimo kampų matematinio modelio sudarymas. Dėl geografinės vietovės, saulės spindėjimo trukmės (tikimybės) toje vietovėje, saulės kolektoriaus eksploatavimo laikotarpio, saulės kolektoriaus su saulės spindulių koncentratoriumi atveju – jų geometrinių parametrų bei saulės spindulių koncentratoriaus paviršiaus atspindžio koeficiento optimalūs orientavimo kampai yra skirtingi. Šiuo metu Lietuvoje saulės kolektoriaus optimalus polinkio kampas nustatytas eksperimentiškai ir kinta nuo 30° iki 60° visiems metams, 15° ... 45° – vasarą ir 30° ... 60° – žiemą. Šiame straipsnyje sudarytas matematinis modelis leidžia tiksliai apskaičiuoti optimalius saulės kolektoriaus bei saulės kolektoriaus su saulės spindulių reflektoriumi orientavimo kampus pasirinktoje geografinėje vietovėje, pasirinkus norimą saulės kolektoriaus eksploatavimo laikotarpį, geometrinius saulės kolektoriaus ir saulės spindulių reflektoriaus parametrus ir kt. Pateikiami optimaliu kampu orientuoto saulės kolektoriaus ir saulės kolektoriaus su saulės spindulių reflektoriumi matematinio modeliavimo palyginamieji sugeriamos saulės spindulinės energijos kiekio rezultatai. Taip pat pateikiami matematinio modeliavimo ir eksperimentinio tyrimo palyginamieji rezultatai.

Raktažodžiai: saulės kolektorius, reflektorius, saulės spindulių reflektorius, optimalūs kampai, saulės spindulinė energija, matematinis modeliavimas

Юргита Григонене, Миндаугас Карнаускас

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ОПТИМАЛЬНЫХ УГЛОВ НАКЛОНА СОЛНЕЧНОГО КОЛЛЕКТОРА И РЕФЛЕКТОРА СОЛНЕЧНЫХ ЛУЧЕЙ

Резюме

В статье представлено математическое моделирование оптимальных углов солнечного коллектора и солнечного коллектора с рефлектором. Оптимальные углы ориентирования бывают разные и зависят от географической местности, длительности солнечного излучения, периода эксплуатации коллектора, в случае солнечного коллектора с рефлектором – от их геометрических параметров и коэффициента отражения поверхности. В настоящее время в Литве экспериментами установлено, что оптимальный угол наклона коллектора находится между 30^0 и 60^0 в течение года, в т. ч. 15° ... 45° – летом и 30° ... 60° – зимой.

С помощью представленной математической модели возможно точно подсчитать углы наклона солнечного коллектора, а также солнечного коллектора с рефлектором в данной географической местности, с выбранным периодом эксплуатации и геометрическими параметрами. Представлены также результаты сравнения математического моделирования и экспериментальных данных.

Ключевые слова: солнечный коллектор, рефлектор, рефлектор солнечных лучей, оптимальные углы, математическое моделирование