Empirical relationship between filtration coefficient and grain size composition of loose rocks

Mykolas Dobkevičius,
Bogdanas Karmazinas,
Petras Klizas

Dobkevičius M., Karmazinas B., Klizas P. Empirical relationship between filtration coefficient and grain size composition of loose rocks. *Geologija*. Vilnius. 2002ž No. 37. P. 49–55. ISSN 1392–110X.

The filtration coefficient values calculated according to widely applied empirical formulae for grain-size composition and porosity of rocks, except for Hazen formula, differ significantly from those determined in the laboratory by means of Kamensky filtrometer and in the field by means of water charged into a drilled well and measured by Nesterov infiltrometer (Dobkevičius, Plankis, 2000). The factor and regression analysis of the data on the relationship of the filtration coefficient to all the grain-size fractions and loose rock porosity enabled to derive the formula for calculations of the filtration coefficient. The article presents the results of the analysis carried out.

Keywords: permeability coefficient, Kamenski filtrometer, granulometry, porosity coefficient

Received 15 February 2002, accepted 25 February 2002

Mykolas Dobkevičius, Petras Klizas. Department of Hydrogeology and Engineering Geology, Vilnius University, M. K. Čiurlionio 21/27, LT-2009 Vilnius, Lithuania

Bogdanas Karmazinas. Geological Survey of Lithuania, S. Konarskio 35, LT-2600 Vilnius, Lithuania

INTRODUCTION

From the times of Hazen (end of the 19th century) up to modern times, quite a few formulae have been created for determination of the filtration coefficient (K) according to grain size composition and porosity of loose rocks. In spite of the facts that the calculations made according to these formulae do not provide satisfactory results, production enterprises use them up to now for their purposes. An analysis of filtration coefficient data calculated by formulae of Hazen, Slichter, Zamarin, Kozeni, Zuenkler, Sauerbrei, Bayer-Schweigel, and Orekhova shows that only Hazen's formula is good enough for the calculation of the filtration coefficient for coarse and medium-grained well-sorted sand deposits (Dobkevičius, Plankis, 2000).

The interval of application of Hazen's formula is very narrow: the effective diameter d10 should be within the range of 0.1–3.0 mm, and the coefficient of heterogeneity d_{10}/d_{60} should vary only from 1 to 5. The rest formulae applied to calculate the filtration coefficient give big errors for loose rocks of Lithuania. In order to get higher accuracy of a relationship of the filtration coefficient to grain-size composition and porosity of rocks, the coefficient values determined with the aid of a Kamensky infiltrometer, grain size data and porosity of 70 rocks sampled in the aeration zone during water recharge experiments in Šiauliai, Klaipėda and Šilutė districts have been analysed.

METHODS FOR DETERMINATION OF RELATIONSHIP BETWEEN FILTRATION COEFFICIENT AND GRAIN-SIZE COMPOSITION

Statistical treatment of the data enabled to derive a formula for calculation of the filtration coefficient as a function of all grain size fractions and rock porosity. For selection of approximating curves as functions of the logarithmic value of the filtration coefficient (lga) from percentages of fractions, the TABLE CURVE software was used, whereas a hidden dependence among the above variables was obtained by the FACTOR ANALYSIS and MULTIPLE REGRESSION programmes. The relationships detected enabled to get new "complex" variables (factors) which, together with the porosity index, were used in a multiple regression model as independent variables presenting lgk as an dependent variable.

At the first stage the statistical treatment was performed only for grain size levels (without fractions 5–10 mm > 10 mm), taking into account the filtration coefficient (lgk) determined in the laboratory.

RESULTS OF FACTOR AND REGRESSION ANALYSIS

A matrix of factor loads determined after rotation of factors in accordance with the Keiser varimax method is given in Table 1 for 7 resulting characteristics for the standardised values, whereas Table 2 shows the common character of the factors and variables. Only the factors with their vectors exceeding 1 were used in the calculation. Table 1 shows that there were two such factors.

Taken together they both provide conditions for the formation of grain size composition by 78.9 percent. Factor 1 (F1) is related mainly to lgk (correlation equals to 0.98) and fraction <0.1 mm, with an impressive negative value of factor load being – 0.963. Table 2 shows that factor F1 joins four fractions (without fr 2–1 and fr 1–0.5 mm) into a correlative association with the filtration coefficient, moreover, the coarse fraction 5–2 mm is of the same sign as that for the factor load of fraction < 0.1 and of an opposite sign for other significant load values of fractions and lgk. That is, factor F1 forms the grain-size composition of fluvioglacial deposits and, hence, makes a basis for a relationship between the lgk and weighing percentages of fractions; some of them (0.5–0.25 mm and 0.25–0.1 mm) increase the values of the lgk, while other fractions (5–2 mm and < 0.1 mm) diminish the lgk (Fig. 1).

Factor F2 shows the absence of a linear correlation between the grain size composition and the filtration coefficient (factor load for F2 is statistically insignificant and equals to –0.029). Such a position of correlative association for factor F2 could mean that the massif under study could contain hidden cyclic components reflecting the process of rock formation with the dynamics of one or several fractions relative to each other. As an example of some samples, the authors present sinusoidal relationships (Fig. 2) between the fractions and the filtration coefficient.

Therefore, applying multiple regression to create an empirical relationship between the lgk and grain-size, factor F2 was not taken into account in the calculations. Since the factors are taken as standardised values calculated according to standardised variables of grain-size composition, for practical convenience of the usage of the empirical relationship, the following actions have been undertaken: Action 1. F1 values were recalculated by means of multiple regression with factual non-standardised values of six fractions taken as independent variables, as well as the lgk was replaced by the porosity index. Regression coefficients and their significance are given in Table 3, and the regression model is shown in Fig. 3.

According to the sign and value of the coefficients and their significance, the degree of impact of each variable can be assessed.

Action 2. Factor F1 data obtained by multiple regression (from the data given in Table 1) were used to calculate the linear relationship between the lgk and F1. Table 4 shows how the complex abstract parameter F1 gives a 92.0-percent explanation (r = = 96) of lgk variations, while Durbin–Watson statistics D = 2.0 testifies to the absence of a significant serial correlation in the model remainders.

The formation of grain size composition can be greatly affected by constantly acting periodical or

Table 1. **Matrix of factor loads**
1 lentelė. **Faktorinės apkrovos matrica**

Variables	Fraction, mm	5–2	2–1	1–0.5	0.5–0.25	0.25–0.1	<0.1	lgk	A part of dispersion
Factors	F1	–0.573	0.012	–0.016	0.574	0.502	–0.963	0.980	0.400
	F2	0.577	0.846	0.871	0.653	–0.716	0.035	–0.029	0.389

Table 2. **Common character of variables and factors**
2 lentelė. **Kintamųjų bendrijos su faktoriais**

Variables	Fraction, mm	5–2	2–1	1–0.5	0.5–0.25	0.25–0.1	<0.1	lgk
Factors	F1	0.3283	0.0002	0.0003	0.330	0.252	0.9265	0.9605
	F1, F2	0.6381	0.7166	0.7590	0.7562	0.7646	0.9277	0.9613

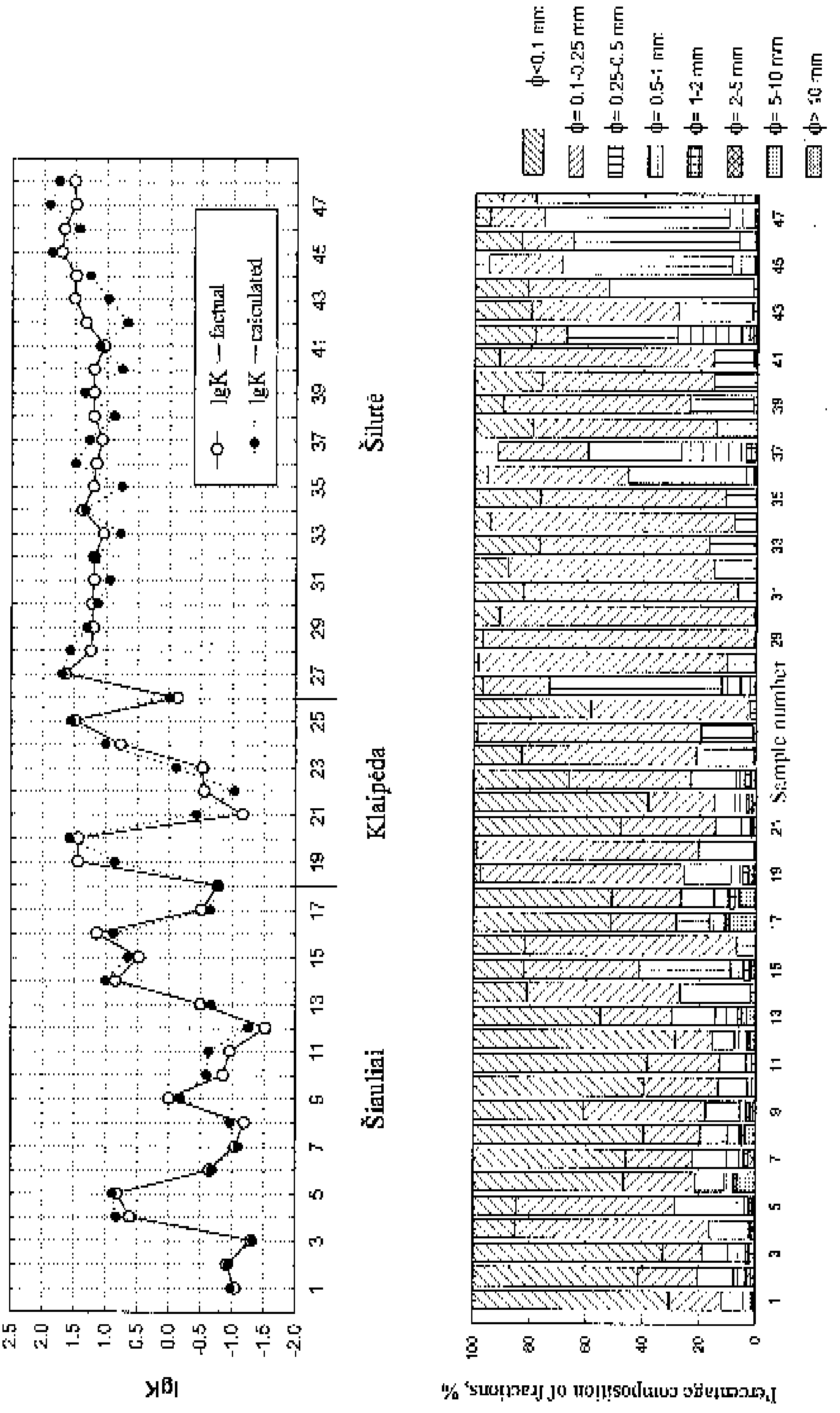

Fig. 1. Comparison of laboratory and model values of the permeability coefficient with the grain size of different rock samples

1 pav. Laboratorinių ir modelinių filtracijos koeficiento verčių palyginimas su granuliometrine atskirų grunto pavyzdžių sudėtimi

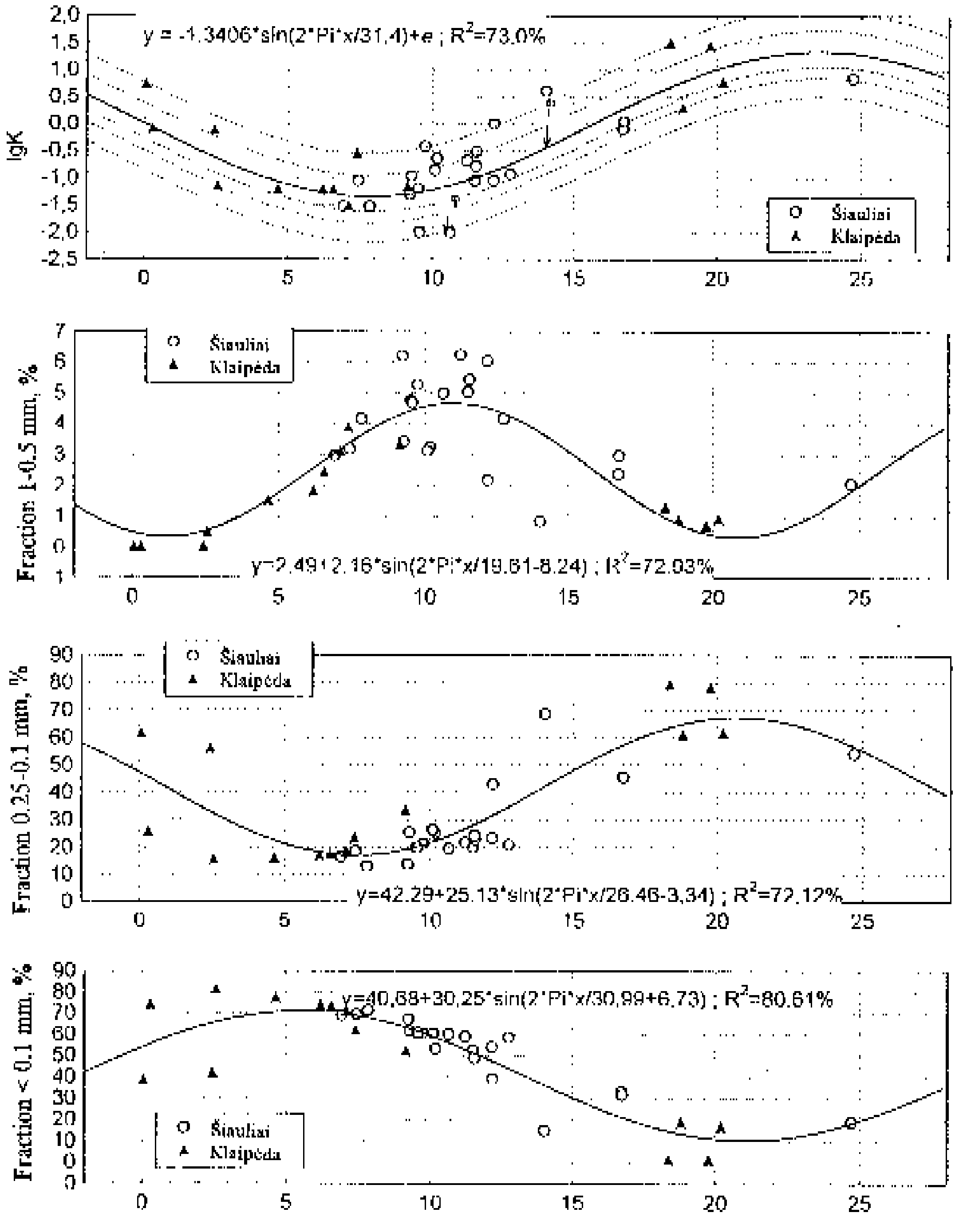

Fig. 2. Dependence of permeability coefficient and composition of different ground fractions on fraction 0.5–0.25 mm
2 pav. Filtracijos koeficiento (lgk) ir grunto skirtingų frakcijų procentinės sudėties priklausomybė nuo 0,5–0,25 mm frakcijos

pulsating forces such as Sun's gravitation-magnetic field pulsation, Earth's natural oscillations, earthquakes, etc. (M. Dobkevičius, B. Karmazinas, 2001), as it is proved indirectly by the presence of factor F2 in the results of factor analysis (see Fig. 2).

The final empirical formula takes into account a 92.0-percent factual dispersion of the lgk (r = 0.96, Durbin–Watson statistics D = 2.05) (Fig. 3):

$$lgk = 0.5094 + 0.99875F_1,$$

$$F_1 = 0.020fr1 + 0.0134fr2 + 0.0233fr3 + 0.0192fr4 + 0.029fr5 - 03295 - 1.161n, \quad (1)$$

where fr1 < 0.1 mm, fr2 = 0.25–0.1 mm, fr3 = 0.5–0.25 mm, fr4 = 1–0.5 mm, fr5 = 2–1 mm, fr6 = 5–2 mm; n is the porosity index. Conditions of the application of the formula are given in Table 5.

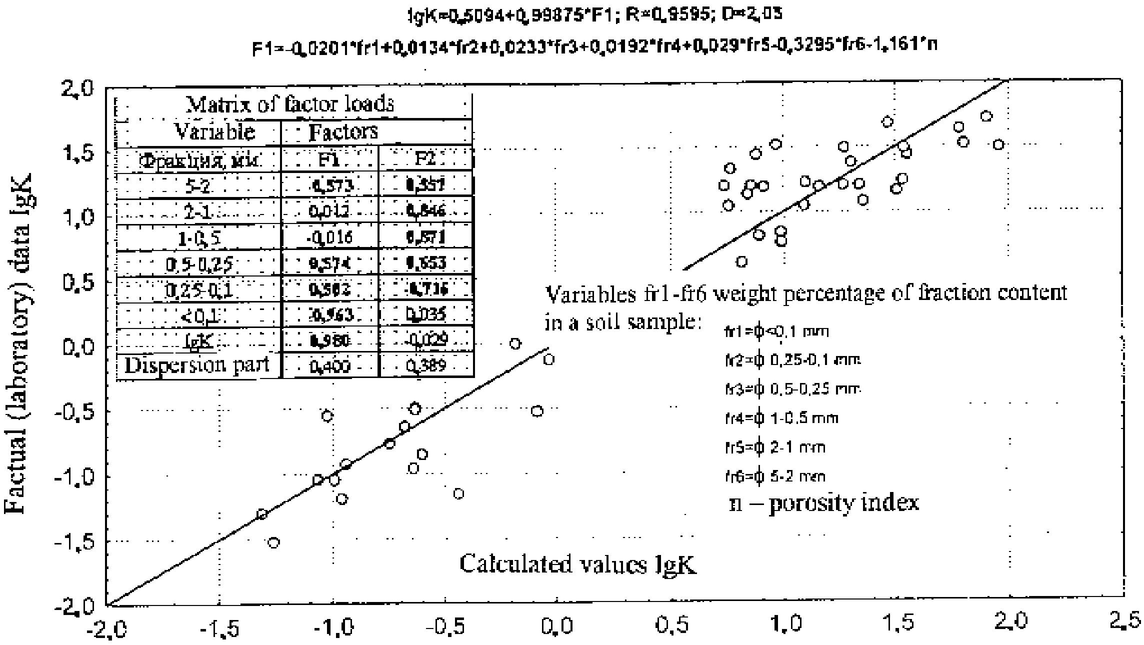

Fig. 3. Calibration line for factual (laboratory) and calculated values of filtration coefficient. Abscissa – calculated lgk values, ordinate – factual (laboratory) data on the lgk
3 pav. Kalibracinė tiesė tarp faktinių (laboratorinių) ir išskaičiuotų filtracijos koeficiento verčių

Table 3. **Results of regression relationship of factor F1 to grain size composition and porosity index (R = 0.9956; R^2 = 0.9912; D = 2.24)**
3 lentelė. **F1 faktoriaus regresinės priklausomybės nuo granuliometrinės sudėties ir poringumo rodiklio rezultatai**

Variables	Regression coefficient	Standard error for regression coefficient	t(41) criterion	P level of significance
Fractions, mm				
5–2	–0.3295	0.0390	–8.4553	0.0000
2–1	0.0290	0.0262	1.1059	0.2752
1–0.5	0.0192	0.0051	3.7940	0.0005
0.5–0.25	0.0233	0.0026	8.9377	0.0000
0.25–0.1	0.0134	0.0027	5.0026	0.0000
<0.1	–0.0201	0.0035	–5.7001	0.0000
n (porosity index)	–1.1610	0.6892	–1.6844	0.0997

Table 4. **Regression relationship between filtration coefficient logarithm and factor F1** (Lgk = 0.5094 + 0.99875*F1; R = 0.9595; R^2 = 0.9207; D = 2.05)
4 lentelė. **Filtracijos koeficiento logaritmo regresinė priklausomybė nuo F1 faktoriaus**

Variable	Regression coefficient	Standard deviation of regression coefficient	t(46) criterion	P level of significance
Free coefficient	0.50940	0.0426	11.9622	0.0000
F1	0.99875	0.0432	23.1060	0.0000

A comparison of the factual and the calculated (formula 1) values of the filtration coefficient is shown in Fig. 1; we see that not all data are conformable. The factual and calculated data correlation coefficient is 0.96. A higher conformity was obtained for the rocks in Šiauliai and Klaipėda districts. This can be explained by several reasons. Firstly, the rocks in the above regions differ in their origin, and this has not been taken into account while constructing the formula. This is confirmed by differences in grain size (Fig. 1), although the lithology of the deposits is similar. Deposit samples taken in Klaipėda and Šiauliai districts exhibit finer fractions (<0.1 mm), while in Šilutė region fractions 0.25–0.1 and 0.5–0.25 mm are more common, hence, the filtration coefficient is higher here. Secondly, while applying factual data for determination of filtration coefficient, the time factor (periodical variations in filtration coefficient in time depending on gravitational-magnetic pulsations in the Sun (see Chapter II) is not taken into account. Thirdly, the filtration coefficient values depend on the accuracy of data on the grain size composition and rock porosity, which presently are determined not very accurately.

Table 5. **Statistical characteristics of deposit grain size composition for empirical relationship (1) constructed to determine the filtration coefficient**
5 lentelė. **Uolienų granuliometrinės sudėties statistinės charakteristikos empirinei filtracijos koeficiento priklausomybei (1) nustatyti**

Fraction, mm	Samples	Average	Confidence interval for an average		Median	Minimum	Maximum	Dispersion	Standard deviation	Standard error
			–95%	+95%						
>10	48	0.01	–	0.04	0	0	0.6	0.008	0.087	0.013
10–5	48	0.91	0.34	1.48	0	0	9.63	3.803	1.950	0.281
5–2	48	0.52	0.33	0.70	0.145	0	2.24	0.416	0.645	0.093
2–1	48	0.79	0.50	1.09	0.455	0	4.8	1.021	1.010	0.146
1–0.5	48	3.74	2.36	5.12	2.935	0	22.7	22.552	4.749	0.685
0.5–0.25	48	20.63	15.47	25.78	14.16	0.6	69.9	315.32	17.757	2.563
0.25–0.1	48	46.19	38.87	53.52	43.01	11	96	635.75	25.214	3.639
<0.1	48	27.21	20.75	33.66	18.89	1.21	71.44	493.95	22.225	3.208
n	48	0.41	0.40	0.42	0.418	0.32	0.46	0.001	0.034	0.005

The grain size data depend also on the determination method. There are several methods applied in practice: mechanical (sieving), silting, pipette, X-ray method, etc. The first method gives grain-size data affected by not only the average diameter of a particle, but also its shape and position on a sieve (Принц, 1933). Moreover, the deposit particles often stick together into lumps which are difficult to break. Some fine particles are attracted by molecular forces to the larger ones.

Performing grain size analysis by the second method, the greatest impact on grain size is exerted by the weight of the particles. The results will not be fully reliable in this case, since smaller particles of more weighty minerals settle more rapidly in water and increase a share of the larger fractions. Calculating the filtration coefficient according to empirical formulae, as a rule, the method of grain-size determination is not taken into account; the results obtained by different methods are not taken into account, either.

For determination of efficient porosity there is no universal methodology, therefore it is rarely determined. In laboratories, usually only general porosity is determined. However, its relationship with efficient porosity is not simple. General porosity approximately is the same, if compared to the efficient one, only in coarse loose rocks or for medium-grained particles. Finer aleurite or clay particles increase the general porosity significantly, but reduce the efficient porosity. This is due to a decrease in size of the pores which can be full of fixed water, and hence filtration at a normal pressure gradients can stop absolutely. The diversity of the relation of general and efficient porosity to pore permeability causes the highest error when determining the filtration capacity by indirect methods. Moreover, the research has shown that determining the weight (w) and density (ρ) of deposits by a thermal method in weighing bottles a too small volume of deposits is used, which finally results in biased data on porosity and hence on the filtration coefficient determined by means of empirical formulae.

CONCLUSIONS

1. Factor and regression analysis showed that fractions < 0.1 mm and 5–2 mm reduce the value of filtration, while the rest fractions increase it.

2. An empirical formula has been constructed for calculation of the filtration coefficient as a function of grain-size and porosity.

References

Dobkevičius M., Plankis M. 2000. Birių uolienų filtracijos koeficiento nustatymas pagal granuliometrinę sudėtį. *Geologija*. ***29***. 100–109.

Добкявичюс М., Кармазин Б. 2001. Зависимость вертикальной фильтрации от изменения гравитационно-магнитного поля Земли. *Геология*. ***35***. 39–47.

Принц Е. 1933. Гидрогеология. Москва: Сельхозиздат. 382 с.

Mykolas Dobkevičius, Bogdanas Karmazinas, Petras Klizas

BIRIŲ UOLIENŲ FILTRACIJOS KOEFICIENTO IR GRANULIOMETRINĖS SUDĖTIES TARPUSAVIO EMPIRINĖ PRIKLAUSOMYBĖ

Santrauka

Nuo Hazeno laikų (XIX a. pabaiga) iki mūsų dienų yra sukurta nemažai empirinių formulių birių uolienų filtracijos koeficientui apskaičiuoti, panaudojant jų granuliometrinės sudėties ir poringumo rodiklio duomenis. Palyginus filtracijos koeficiento reikšmes, nustatytas laboratorijoje Kamenskio filtrometru ir lauke įpylimų į šurfus S. Nesterovo infiltrometru, nustatyta, kad, išskyrus Hazeno formulę, kitos formulės Lietuvos uolienų filtracijos koeficientui apskaičiuoti duoda labai dideles paklaidas. Be to, Hazeno formulė gali būti taikoma tik labai siauram grūdelių dydžio ir efektyvaus skersmens intervalui, faktiškai, tik vidutingrūdžiui ir stambiagrūdžiui gerai išrūšiuotam smėliui.

Filtracijos koeficiento ryšio su birių uolienų granuliometrinėmis frakcijomis ir poringumu pagrindu, panaudojus faktorinę ir regresinę analizes ir įvertinus 6 granuliometrinės sudėties frakcijas bei poringumo rodiklio duomenis, buvo gauta nauja empirinė formulė.

Миколас Добкевичюс, Богданас Кармазинас, Пятрас Клизас

ЭМПИРИЧЕСКАЯ ЗАВИСИМОСТЬ МЕЖДУ КОЭФФИЦИЕНТОМ ФИЛЬТРАЦИИ И ГРАНУЛОМЕТРИЧЕСКИМ СОСТАВОМ СЫПУЧИХ ПОРОД

Резюме

Со времён Хазена (конец XIX в.) до наших дней создано немало эмпирических формул для определения коэффициента фильтрации сыпучих пород по данным их гранулометрического состава и пористости. Сопоставление значений коэффициента фильтрации, определённых в лаборатории с помощью фильтрометра Каменского и в поле способом наливов воды в шурфы используя инфильтрометр Н. Нестерова, показало, что эти формулы, за исключением формулы Хазена, совершенно не пригодны для расчёта коэффициента фильтрации для пород Литвы. Кроме того, эта формула имеет очень узкий интервал применения и пригодна лишь для расчёта коэффициента фильтрации по существу только для крупно- и среднезернистых, хорошо отсортированных песков.

На основе данных анализа связи коэффициента фильтрации с фракциями гранулометрического состава и пористостью сыпучих пород с помощью факторного и регрессионного анализа была получена новая эмпирическая формула для его расчёта используя данные 6-и фракций гранулометрического состава и показатель пористости.