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ANOMALOUS DIFFRACTION OF IDLER BEAM GENERATED IN OPTICAL PARAMETRIC AMPLIFIER

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The diffraction of idler beam generated in optical parametric amplifier operating at low parametric gain is analysed. It is revealed that diffraction properties of idler beam under propagation in the free space is significantly different in comparison with the properties of Gaussian beam.

Keywords: nonlinear optics, diffraction, optical parametric amplifier

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1. Introduction

Propagation of intense light beam in nonlinear medium is governed by interplay of diffraction and nonlinearity. The refractive index of isotropic medium is modified by intense beam. As a result, the diffraction can be compensated, and self-focusing of the beam occurs. Anomalous behaviour of diffraction in a medium with quadratic nonlinearity was predicted in [1, 2]. It was shown that a beam injected into an optical parametric amplifier (OPA) can acquire under diffraction a converging wave front at low energy exchange with a pump beam. Simultaneous mutual focusing of interacting beams in nonlinear crystal is possible at strong energy exchange. The wave vectors of signal and idler waves in the nonlinear crystal due to the action of the pump wave are modified in comparison with the linear propagation. In OPA which operates near degeneracy the diffraction of the signal beam can be effectively suppressed [3, 4]. At degeneracy the idler beam is entirely diffraction-free, and perfect spatial localization of an idler beam is obtained.

In what follows we analyse the diffraction of idler beam generated in OPA and reveal that its behaviour under propagation in the free space can be significantly different in comparison with propagation of a Gaussian beam.

2. Theoretical treatment

We shall analyse the parametric interaction of the quasimonochromatic light beams in a nonlinear crystal neglecting depletion of the pump wave. The corresponding truncated equations of nonlinear optics are [5]

$$\frac{\partial A_1}{\partial z} = -\frac{\mathrm{i}}{2k_1}\triangle_\perp A_1 + \mathrm{i}\sigma_1 A_3 A_2^*\,,$$
$$\frac{\partial A_2}{\partial z} = -\frac{\mathrm{i}}{2k_2}\triangle_\perp A_2 + \mathrm{i}\sigma_2 A_3 A_1^*\,, \qquad (1)$$

where A_1, A_2, and A_3 are complex amplitudes of signal, idler, and pump beams, respectively. For simplicity, we suppose that the pump beam is a plane wave with an amplitude $A_3 = a_3\exp(\mathrm{i}\varphi_3)$, and noncritical phase matching of interacting waves takes place. $\triangle_\perp = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ is a transverse Laplacian operator, k_m and σ_m are wave vector and nonlinear coupling parameter, respectively, of signal ($m = 1$) and idler ($m = 2$) beams, z is a longitudinal coordinate. Assuming axial symmetry for signal and idler beams, further we take into consideration the angular spectra of signal and idler waves $S_1(\beta)$ and $S_2(\beta)$, where β is angular frequency. Carrying out two-dimensional Fourier transform of Eqs. (1) we find the solutions for the angular spectra $S_1(\beta)$ and $S_2(\beta)$ [4]:

$$S_1 = S_0(\beta)\exp(\mathrm{i}ql)\left[\cosh(pl) + \mathrm{i}\xi\frac{\sinh(pl)}{p}\right],$$

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$$S_2 = \mathrm{i}\sigma_2 A_3 S_0^*(-\beta)\exp(-\mathrm{i}ql)\frac{\sinh(pl)}{p}\,, \tag{2}$$

where $q = \beta^2/4 \cdot (1/k_1 - 1/k_2)$, $\xi = \beta^2/4 \cdot (1/k_1 + 1/k_2)$, $p = \sqrt{\Gamma^2 - \xi^2}$, $\Gamma = \sqrt{\sigma_1\sigma_2}a_3$ is the parametric gain factor, and l is nonlinear crystal length. It was assumed that at $z = 0$ in OPA $S_1 = S_0(\beta)$ and $S_2 = 0$. In the case of nondegenerate parametric interaction an idler beam can acquire a converging spherical wave front ($k_2 > k_1$, $q > 0$) as well as a diverging one ($k_2 < k_1$, $q < 0$) [6].

Further we shall analyse the propagation of an idler beam in the free space behind the OPA which operates near degeneracy. Then $k_1 \approx k_2 = k$, $\sigma_1 \approx \sigma_2 = \sigma$, $q \approx 0$, $\xi \approx \beta^2/(2k)$, $p = \sqrt{\Gamma^2 - \beta^4/(4k^2)}$, $\Gamma \approx \sigma a_3$. In this case the 2nd equation of Eqs. (2) can be rewritten in the form

$$S_2 = \mathrm{i}\sigma A_3 l S_0^*(-\beta)F(\beta)\exp\left(\frac{\mathrm{i}\beta^2 z'}{2k_0}\right), \tag{3}$$

where

$$F(\beta) = \frac{\sinh(pl)}{pl} \tag{4}$$

is OPA transfer function, which determines the band of angular frequencies, and factor $\exp[\mathrm{i}\beta^2 z'/(2k_0)]$ describes the propagation of an idler beam in the free space behind the crystal at $z' = z - l > 0$. Here $k_0 = k/n$ and $n = n_2 \approx n_1$ is a refraction index of the nonlinear crystal. We assume that at the input of OPA ($z = 0$) the signal beam is Gaussian with a plane wavefront. In this case the amplitude of the signal beam at the input boundary of the crystal can be written as $A_{10} = a_{10}\exp(-r^2/d_0^2)$, where r is a radial coordinate, and d_0 is the beam radius. Carrying out a Fourier transform we obtain an angular spectrum of the signal beam at the input of OPA

$$S_0(\beta) = \pi d_0^2 a_{10}\exp\left(-\frac{\beta^2 d_0^2}{4}\right). \tag{5}$$

By use of Eq. (5) we find

$$S_2 = \mathrm{i}\pi d_0^2 \Gamma l a_{10}\exp(\mathrm{i}\varphi_3) \times \exp\left(-\frac{\beta^2 d_0^2}{4} + \mathrm{i}\frac{\beta^2 z'}{2k_0}\right)F(\beta)\,. \tag{6}$$

Then, the complex amplitude $A_2(r)$ of the idler beam can be found as two-dimensional Fourier transform of Eq. (6),

$$A_2(r) = \frac{1}{2\pi}\int_0^\infty \beta S_2(\beta)J_0(\beta r)\,\mathrm{d}\beta\,, \tag{7}$$

where $J_0(\beta r)$ is zeroth-order Bessel function of the first kind. It follows that

$$A_2(r) = \frac{\mathrm{i}}{2}\Gamma l d_0^2 a_{10}\exp(\mathrm{i}\varphi_3) \times \int_0^\infty \beta\exp\left(-\frac{\beta^2 d_0^2}{4} + \mathrm{i}\frac{\beta^2 z'}{2k_0}\right)F(\beta)J_0(\beta r)\,\mathrm{d}\beta\,. \tag{8}$$

It should be pointed out that the idler beam in degenerate OPA is diffraction-free (see Eq. (6) at $z' = 0$), and perfect spatial localization of the idler beam in the crystal can be obtained [4]. Equation (8) can be rewritten as

$$A_2(\rho) = \mathrm{i}\Gamma l a_{10}\exp(\mathrm{i}\varphi_3)f(\rho, s)\,, \tag{9}$$

where

$$f(\rho, s) = \frac{1}{b}\int_0^\infty \exp\left(-\frac{y}{b} + \mathrm{i}sy\right)\frac{\sinh\sqrt{\Gamma^2 l^2 - y^2}}{\sqrt{\Gamma^2 l^2 - y^2}}J_0(\rho\sqrt{y})\,\mathrm{d}y\,, \tag{10}$$

and $y = \beta^2 l/2k$, $s = z'n/l$, $\rho = \frac{2r}{\sqrt{b}d_0}$, $b = l/L_d$; here $L_d = kd_0^2/2$ is a Rayleigh range of a signal beam in the crystal.

3. Difference-frequency generation

At $\Gamma l \ll 1$ the parametric interaction for idler beam can be treated as difference-frequency generation (DFG), and OPA transfer function $F(\beta)$ takes a form

$$F(\beta) \approx \frac{\sin\left(\frac{\beta^2 l}{2k}\right)}{\left(\frac{\beta^2 l}{2k}\right)} = \frac{\sin y}{y}\,. \tag{11}$$

First in this case we shall analyse the variation of axial amplitude $|A_2(0)|$ of idler beam during its propagation behind the crystal. We obtain

$$\frac{|A_2(0)|}{a_{10}} = \Gamma l|f(0, s)| = \frac{\Gamma l}{b}\left|\int_0^\infty \exp\left(-\frac{y}{b} + \mathrm{i}sy\right)\frac{\sin y}{y}\,\mathrm{d}y\right|. \tag{12}$$

At the output of OPA ($s = 0$) we find

$$\frac{|A_2(0)|}{a_{10}} = \Gamma l\frac{\arctan b}{b}\,. \tag{13}$$

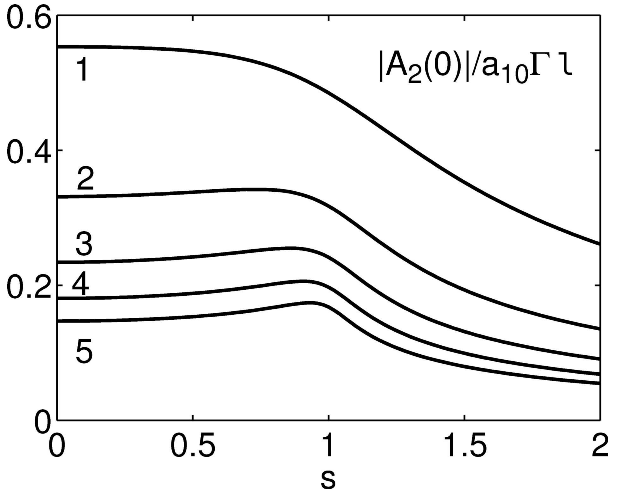

Fig. 1. Difference-frequency generation. Variation of axial amplitude of idler beam under propagation behind OPA in the free space. Parameter b: 2 (*1*), 4 (*2*), 6 (*3*), 8 (*4*), 10 (*5*).

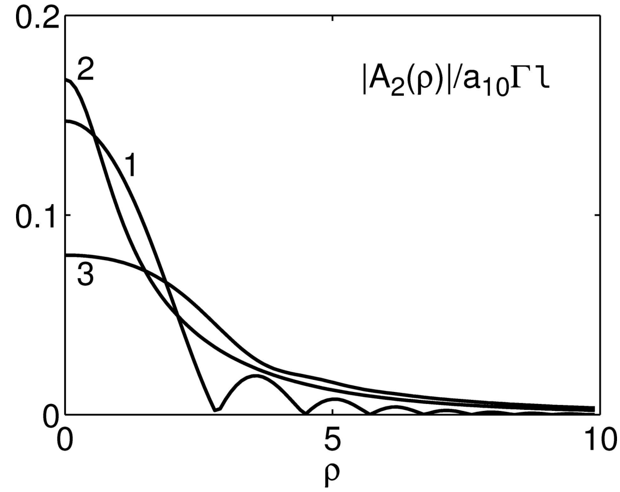

Fig. 2. Difference-frequency generation. Profile of idler beam under propagation behind OPA in the free space. Parameter $b = 10$, parameter s: 0 (*1*), 1 (*2*), 1.5 (*3*).

At $b \ll 1$ ($l \ll L_d$) the axial amplitude of idler beam increases linearly with a crystal length, $|A_2(0)|/a_{10} = \Gamma l$. If $b \gg 1$ ($l \gg L_d$), then axial amplitude of idler beam at the output of OPA is determined by Rayleigh range of the signal beam, $|A_2(0)|/a_{10} \approx \pi/2 \cdot \Gamma L_d$. In general, an integration in Eq. (12) yields:

$$\frac{|A_2(0)|}{a_{10}} = \tag{14}$$

$$\frac{\Gamma l}{2b}\left|\arctan\frac{2b}{(s^2-1)b^2+1} + j\pi + \frac{\mathrm{i}}{2}\ln\frac{1+b^2(s+1)^2}{1+b^2(s-1)^2}\right|,$$

here $j = 0$ at $(s^2-1)b^2+1 \geq 0$ and $j = 1$ if $(s^2-1)b^2+1 < 0$. The analysis of Eq. (14) shows that just behind the crystal the axial amplitude of idler beam increases due to diffraction when

$$\frac{\arctan b}{b} < 0.5\,. \tag{15}$$

A solution of inequality (15) is $b > 2.3$.

The dependence of an axial amplitude of the idler beam on normalized longitudinal coordinate $s = z'n/l$ under propagation behind OPA in the free space obtained by use of Eq. (14) is presented in Fig. 1 for various values of b. We note that for broad signal beams ($b \ll 1$, $l \ll L_d$) OPA transfer function $F(\beta)$ is $\sin(y)/y \approx 1$, and the integration in Eq. (12) yields

$$\frac{|A_2(0)|}{a_{10}} = \frac{\Gamma l}{\sqrt{1+\left(\frac{z'n}{L_d}\right)^2}}\,. \tag{16}$$

So, for $b \ll 1$ the variation of the axial amplitude of the idler beam under propagation is typical for Gaussian beam. Obviously, with increase of parameter b (Fig. 1) the variation of axial amplitude of idler beam under diffraction is quite different in comparison with a variation typical for Gaussian beam. For $b > 2.3$, the axial amplitude behind the crystal slowly increases, reaches a maximum at $s \approx 1$ ($z' \approx l/n$), and afterwards rapidly decreases. An explanation of this phenomenon is quite simple. The OPA transfer function (11) can be written as

$$F(\beta) \propto \frac{\exp\left(\frac{\mathrm{i}\beta^2 l}{2k}\right) - \exp\left(-\frac{\mathrm{i}\beta^2 l}{2k}\right)}{\frac{\beta^2 l}{2k}}\,. \tag{17}$$

It means that the idler beam at the output of OPA can be treated as a superposition of two beams, one with converging wave front and another with diverging one. In this case the focusing of the beam with converging wave front is observed behind the crystal, causing the increase of axial amplitude of idler beam.

The profile of the idler beam under diffraction behind OPA obtained by numerical integration in (10) is presented in Fig. 2. Obviously, at the output of OPA ($s = 0$) the beam profile is non-Gaussian (curve *1*). It should be pointed out that the complicated profile of the idler beam in this case is caused by transfer function of OPA $F(\beta)$, which modifies the Gaussian profile of angular spectrum of the signal beam

$$\frac{S_2(y)}{S_2(0)} = \exp\left(-\frac{y}{b}\right)\frac{\sin y}{y}\,, \quad y = \frac{\beta^2 l}{2k}\,, \tag{18}$$

see Eq. (10) and Fig. 3. Under diffraction the beam amplitude on the axis increases, beam width gradually decreases, and oscillating parts of the beam profile

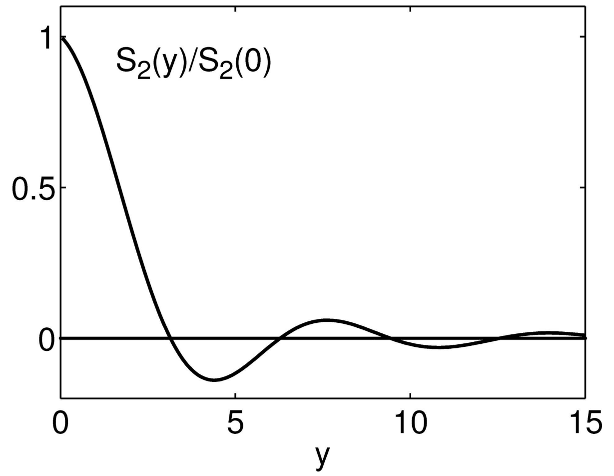

Fig. 3. Difference-frequency generation. Angular spectrum of idler beam at the output of OPA. $y = \beta^2 l/(2k)$, $b = 10$.

disappear (Fig. 2, curve *2*). Afterwards, strong diffraction takes place (Fig. 2, curve *3*).

We note that some diffraction characteristics of idler beam produced in OPA are similar to diffraction characteristics of super-Gaussian and flat-topped Gaussian beams [8–10]. For example, under propagation the axial amplitude of super-Gaussian beam with envelope $\exp\left[-(r/d_0)^{2m}\right]$ first increases at $m = 2$ and afterwards decreases as idler beam generated in OPA.

4. Parametric amplification

The diffraction properties of idler beam behind the crystal essentially depend on the parametric gain factor Γ, as well as on crystal length l. Typical dependences of axial amplitude of idler beam on normalized propagation distance s are shown in Fig. 4 for various values of Γl and b. For rather small values of product Γl (Fig. 4(a, b)) the slow variation of axial amplitude behind the crystal takes place at (a) $b \geq 4$ and (b) $b \geq 10$ up while distance $s \leq 1$, as in the case of DFG, see Fig. 1. The increase of axial amplitude of idler beam behind the crystal now is observed for larger values of parameter b in comparison with DFG. Thus, the idler beam formed in OPA at small parametric gain still preserves its anomalous diffraction properties under propagation in the free space.

In the case of large parametric gain ($\Gamma l = 5$, Fig. 4(c)) the variation of axial amplitude of idler beam behind OPA is quite different and is similar to the variation of axial amplitude of Gaussian beam under diffraction. That is the result of broad band of angular frequencies of OPA existing at large parametric gain [7]. In this case the width of the transfer function $F(\beta)$ exceeds the width of the angular frequencies of the signal

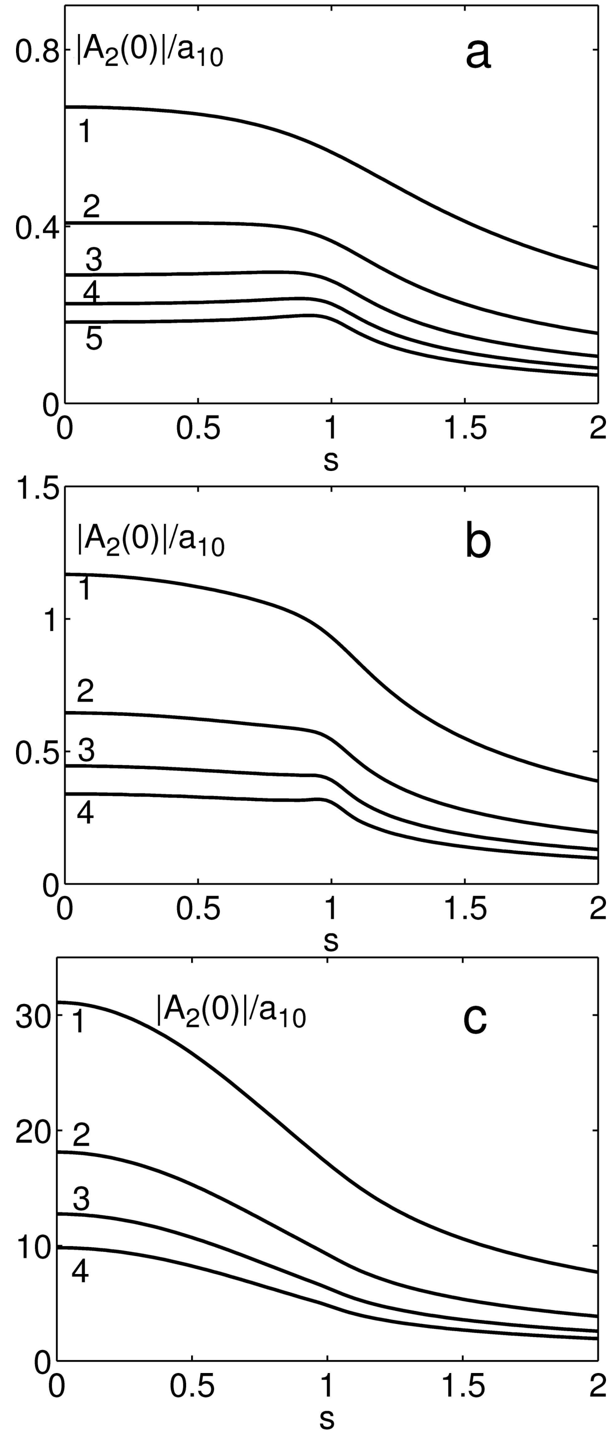

Fig. 4. Parametric amplification. Variation of axial amplitude of idler beam under propagation behind OPA in the free space. (a) $\Gamma l = 1$, $b = 2$ (*1*), 4 (*2*), 6 (*3*), 8 (*4*), 10 (*5*). (b) $\Gamma l = 2$, $b = 5$ (*1*), 10 (*2*), 15 (*3*), 20 (*4*). (c) $\Gamma l = 5$, $b = 5$ (*1*), 10 (*2*), 15 (*3*), 20 (*4*).

beam, and for this reason the idler beam at the output of OPA is Gaussian-like.

5. Conclusions

The diffraction of idler beam generated in OPA is analysed under propagation in the free space. It is shown that at low parametric gain the diffraction of idler beam behind the nonlinear crystal is quite different in comparison with the diffraction of Gaussian beam. The beam width of the idler beam decreases. Its axial amplitude behind the crystal increases, reaches maximum at the distance $\approx l/n$, and afterward rapidly decreases. This phenomenon is more distinct with increase of the ratio $b = l/L_d$. That is caused by transfer function of OPA which modifies the Gaussian profile of angular spectrum of signal beam. In the case of large parametric gain the diffraction of idler beam behind OPA is similar to the diffraction of Gaussian beam.

The revealed diffraction properties of the idler beam generated in OPA should be taken into account due to its applications in laser spectroscopy, optical communications, etc.

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ANOMALI ŠALUTINĖS BANGOS, GENERUOJAMOS OPTINIAME PARAMETRINIAME STIPRINTUVE, DIFRAKCIJA

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Santrauka

Išanalizuota anomali šalutinės bangos, generuojamos optiniame parametriniame stiprintuve, difrakcija mažo stiprinimo atveju. Parodyta, kad šalutinės bangos, sklindančios laisvoje erdvėje, difrakcinės savybės esmingai skiriasi nuo Gauso pluoštui būdingų savybių.